

# Combining SAT solving with Integer Programming for Inductive Verification of Lustre Programs

3rd December 2004

# Outline

## 1 Introduction

- The Lustre programming language
- Temporal induction
- Propositional logic

## 2 Verification

- The decision procedure (SAT + Integer Programming)
- Variants of the basic algorithm

## 3 Analysis

- Test plan
- Comparison with Luke

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# Lustre

```
node Counter ( X : bool ) returns ( C : int );  
    var PC : int;
```

```
let
```

```
    PC = 0  $\rightarrow$  pre C;  
    C = if X then PC + 1 else PC;
```

```
tel
```

```
node Prop( X : bool ) returns ( OK : bool );
```

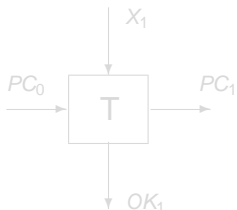
```
let
```

```
    OK = Counter( X )  $\geq$  0;
```

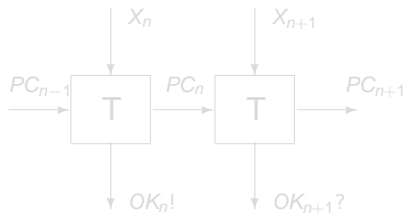
```
tel
```

# Verification by induction

- Prove property valid in initial time point



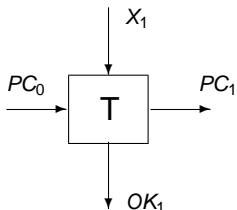
- Assume property valid at time  $n$ , prove property valid at time  $n + 1$



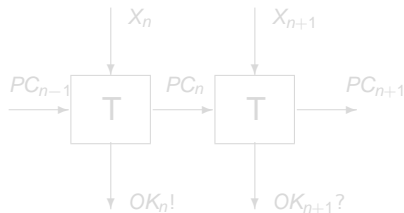
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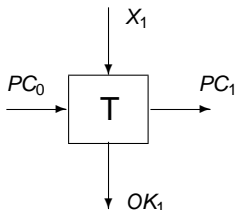
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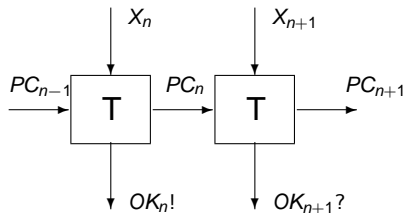
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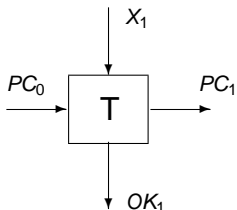


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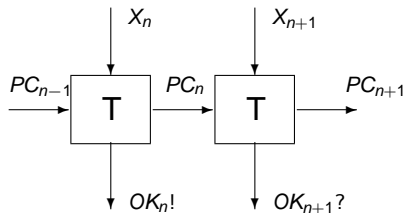


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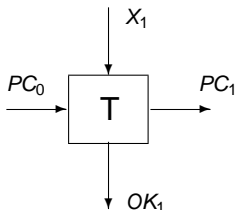
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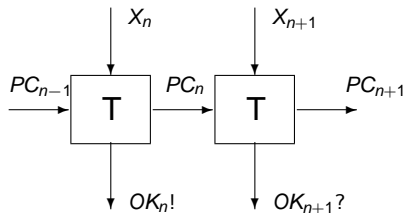
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# Propositional logic

## Example

$\{p, q\}$

## Short introduction

- A clause is a set of literals. At least one literal must be true.
- A formula is a set of clauses. All clauses must be true.

# Propositional logic

## Example

$$\begin{aligned} &\{p, q\} \\ &\{p, \neg q, r\} \\ &\{\neg q, \neg r\} \end{aligned}$$

## Short introduction

- A clause is a set of literals. At least one literal must be true.
- A formula is a set of clauses. All clauses must be true.

# SAT solving

## Example

$\{p, q\}$

$\{p, \neg q, r\}$

$\{\neg q, \neg r\}$

## Search for a satisfying variable assignment

- Choose a variable, and assign at value to it
- Infer consequences
- Repeat until all variables assigned, or a *conflict* found

# SAT solving

## Example

$\{\mathbf{p}, q\}$

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$p = \perp$

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$$p = \perp$$

$$q = \top$$

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$p = \perp$

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$r = \top$  **and**  $\perp??$

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$p = \perp$

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## Search for a satisfying variable assignment

- Analyze reason for conflict
- Add *conflict clause*
- Backtrack and continue

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$$\begin{aligned} p &= \perp \\ q &= \top \\ r &= \top \text{ and } \perp?? \end{aligned}$$

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# A small example

The formula in CNF

## A simple counter

```
node Counter() returns ( OK : bool );
    var C : int;
let
    C = 0  $\rightarrow$  pre C + 1;
    OK = C  $\geq$  0;
tel
```

- Translate to logic
- Assume property invalid
- Is there a variable assignment satisfying the formula?

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The formula in CNF

$$\begin{aligned} &\{ C_1 \leq 0 \} \\ &\{ C_1 \geq 0 \} \\ &\{ \neg OK_1, C_1 \geq 0 \} \\ &\{ OK_1, C_1 \leq -1 \} \end{aligned}$$

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## Step 1: Create *in-place* variables

Create a fresh propositional variable for each constraint

$$\begin{aligned} p_1 &\mapsto C_1 \leq 0 \\ p_2 &\mapsto C_1 \geq 0 \\ p_3 &\mapsto C_1 \geq 0 \\ p_4 &\mapsto C_1 \leq -1 \end{aligned}$$

And replace all constraints with their in-place variable.

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## Step 2: Run through SAT solver

A SAT model is returned

$p_1 = \top$   
 $p_2 = \top$   
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 $\text{OK}_1 = \perp$

Create a constraint problem based on the in-place variables.

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## Step 3: Solve constraint problem

Run constraint problem through ILP solver

- (1)  $C_1 \leq 0$
- (2)  $C_1 \geq 0$
- (3)  $C_1 < 0$
- (4)  $C_1 \leq -1$

Constraint 2 and 4 contradict each other.  
Add explanation to SAT problem. Goto step 2.

# The basic algorithm

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The formula is unsatisfiable

$\Rightarrow$

The original formula is unsatisfiable.

The property is valid in first time point



# The basic algorithm

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# The algorithm

$\varphi$  is a propositional + constraints formula

**loop**

$\mathbb{I}_p \leftarrow \text{Psat}(\varphi)$

**if**  $\mathbb{I}_p = \emptyset$  **then**

**return** *unsatisfiable*

**else**

$C \leftarrow \text{generate}(\varphi, \mathbb{I}_p)$

**if**  $\text{Csat}(C)$  **then**

**return** *satisfiable*

**else**

$\varphi \leftarrow \varphi \cup \text{explain}(C)$

**end if**

**end if**

**end loop**

# Other ideas

- Check *partial* SAT models  
 Everytime the SAT solver assigns an in-place variable, check the constraint problem generated by the set of assigned in-place variables.
- Several methods of creating explanations  
 Several algorithms exist. Finding multiple explanations.
- Preprocessing  
 Find contradictions in the set of constraints before the decision procedure starts.
- Faster (incomplete) integer programming procedure  
 Use a cheap procedure that can find the most commonly occuring contradictions in constraint problems.

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# Rantanplan

- Implements all ideas outlined here
- Based on Luke
- SAT solver changed to MiniSat
- Integer programming package GLPK

# Test plan

## Aim

- What combinations of ideas work well?
  - How do these ideas compare to Luke and NBAC?
- 
- Find “good” combinations of ideas
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# Test suite

Every test should be verifiable by every tool in the tests.

- The test suite consists of 137 tests.
- Some of these are invalid properties. Can not be verified in NBAC.
- Some have too weak properties. Can not be verified in Rantanplan.
- Some used unbounded integers. Can not be verified in Luke.
- Some uses modulo. Can not be verified in NBAC.
- Some generates constraint problems where branch-and-bound does not terminate. Can not be verified in Rantanplan.

We are left with 72 tests.

# Test suite

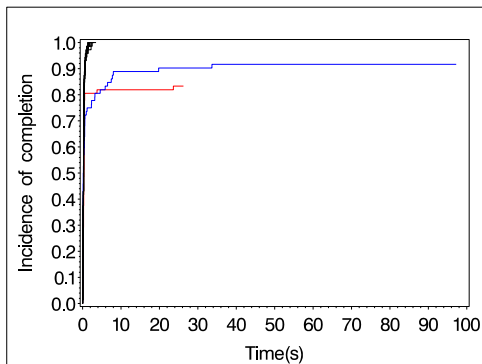
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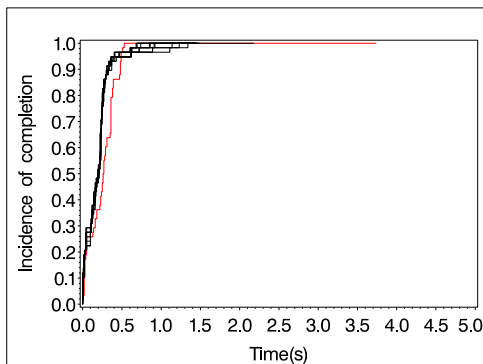
# Comparisons

## Tests of the 11 best variants against Luke and NBAC



# Comparison with Luke

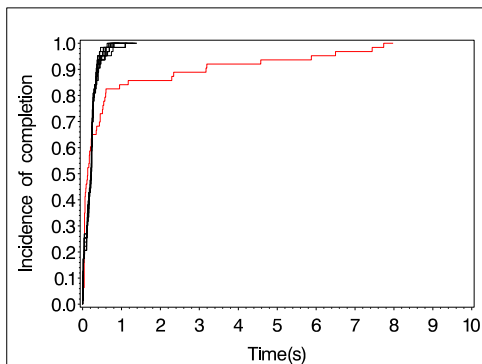
Tests with execution time  $> 10$ s in Luke removed  
(58 remaining)





# Comparison with NBAC

Tests with execution time  $> 10$ s in NBAC removed  
(63 remaining)



# Summary

- Rantanplan competitive on the test suite used here
- The branch-and-bound algorithm is incomplete
- For longer induction depth (e.g. invalid properties w. long counter-examples), Luke outperforms Rantanplan
- Outlook
  - Complete integer programming procedure
  - Improvements for larger induction depths
  - Invariant strengthening