From Sound and Complete to Approximate Deduction

Fabio Massacci*
Dip. di Informatica e Sistemistica, Univ. di Roma "La Sapienza",
via Salaria 113, 00198 Roma, Italy,
e-mail: massacci@dis.uniroma1.it

Abstract. The computational complexity of reasoning in classical and non-classical logics makes traditional deduction not feasible in practice. This paper advocates the introduction of *approximate proofs* within automated deduction for classical and non-classical logics and a corresponding intuition of *superficial semantics* to overcome this limitation.

Recent and past years have seen a vigorous development of automated deduction procedures for classical and non-classical logics. Calculi for logics for knowledge and belief, actions and programs, concurrency and temporal reasoning, description logics for knowledge representation have been developed (e.g. see [2, 3, 6, 7, 10, 11]) and corresponding systems implemented. Classical logic has gone far beyond and competitions between provers are now common.

However, the applicability of deduction methods to engineering is hindered by their computational complexity: even propositional logic is NP complete and other decidable logics are PSPACE or EXPTIME. This computational obstacle has lead to heuristics and tactics to prune the search spaces or to techniques based on knowledge compilation. Still, if one looks at tables of experimental articles in conference proceedings or scientific journals, beside "theorem" and "non-theorem", a common entry is "time-out". As pointed out by Levesque [4], "human reasoners do not time out" and engineers do it even less: after computing resources have been consumed (and paid for), some indicative result is anyway expected for action.

To turn around this obstacle, approximation techniques have been developed with a particular attention to problems in NP (e.g. see [1, 8, 9]). This has led to a theoretical classification of approximability [1] and an experimental analysis of satisfiability which identifies easy and hard "average" cases [9]. Most of these approaches [1, 9] focus on partial satisfiability (e.g. find an assignment which satisfy at least 90% of the clauses) and also [8] bases its analysis of entailment on the underlying satisfiability problem.

An analysis of this research area reveals that, although there are plenty of studies on *approximating satisfiability*, there is little work on the dual problem, the core of automated reasoning: *approximating deduction*. In the opinion of the author, the road from deduction to engineering passes through approximation, and thus *how can a theorem be approximately valid*?

The key point is that satisfiability requires a *witness*, which satisfies all constraints. An approximate witness only satisfies part of them. On the contrary validity requires a *proof* that a witness for a counterexample does not exists (if we apply refutational theorem proving). Thus a theorem can be approximately valid if we find an *approximate proof* with a suitable calculus. Still, the basics of deduction tell us that we expect a calculus to be sound and complete w.r.t. a given semantics. So, we also need a *superficial semantics* which approximate the original semantics (e.g. superficial Kripke models for modal logics) to establish soundness and completeness for approximate proofs.

This seems just to shift the problem: *how can we devise a superficial semantics*? Consider the simple case of propositional logic (two or multi-valued): *classical deduction gives the same importance to all propositions*. On the contrary not all proposition may be meaningful for an

*Current address: Computer Laboratory, University of Cambridge, New Museums Site, Pembroke Street, Cambridge CB2 3QG, UK, e-mail: Fabio.Massacci@cl.cam.ac.uk.

user or he may regard them trivial and therefore *meaningless and trivial propositions (*according a particular user for a particular query*) should be neglected by superficial semantics.* A typical example can be a person browsing a dictionary [8]: if he does not understand a sentence φ, because it contains unknown names p, q... he will not start a theorem prover to check whether φ is a tautology; he will just ignore it and go ahead.

This approach has been carried in the framework of contextual reasoning [5] where a tableaux calculus for approximate proofs and a superficial semantics have been devised, following the intuitions of [4, 8]. The user can then establish a vocabulary of meaningful atomic propositions w.r.t. the context, so that $p \vee \neg p$ may not be valid if, in the given context, p is meaningless, whereas $q \vee \neg q$ is still valid for meaningful q.

The case is worth generalizing to modal and temporal logics: why could not a user only care about the beliefs of the most important agents and disregard the others?

Hence it may be worth fixing some minimal requirements for an approximate calculus:

1. possibility for the user to establish which part of the knowledge base must behave classically (e.g. the meaningful literals or the relevant agents) and which must be approximated;
2. an appealing computational behavior, so that exponential resources are only spent on the relevant part of the KB whereas the irrelevant one is dealt with in polynomial time;
3. incremental computation, to be tuned according the desired degree of approximation, still converging to the original calculus (first order, modal, temporal etc.);
4. incorporation of both sound and complete approximations of the original calculus but also of other intermediate[1] forms;
5. a natural mapping of the partition into relevant and approximable parts of the language onto the original semantics[2];
6. finally, the soundness and completeness of the calculus w.r.t. the superficial semantics.

With these requirements, approximate proofs can make logics for reasoning about knowledge, actions or time more appealing for engineers, while keeping their firm mathematical background.

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[1] For instance sound wrt certain sentences and complete w.r.t. others.

[2] When everything is relevant we should get back the initial semantics of the non approximate calculus