Entity Resolution with Crowd Errors

Vasilis Verroios, Hector Garcia-Molina
Stanford University
{verroios, hector}@stanford.edu

***Abstract*—Given a set of records, an ER algorithm finds records that refer to the same real-world entity. Humans can often determine if two records refer to the same entity, and hence we study the problem of selecting questions to ask error-prone humans. We give a Maximum Likelihood formulation for the problem of finding the "most beneficial" questions to ask next. Our theoretical results lead to a lightweight and practical algorithm, bDENSE, for selecting questions to ask humans. Our experimental results show that bDENSE can more quickly reach an accurate outcome, compared to two approaches proposed recently. Moreover, through our experimental evaluation, we identify the strengths and weaknesses of all three approaches.**

I. Introduction

Entity resolution (ER) detects records that represent the same real-world entity. Computer algorithms are often used for ER, but in many cases where records contain images or natural language, humans can more accurately tell if two records represent the same real-world entity. Google, Bing, and Facebook use this approach to create summary records for entities in web search or resolve duplicates for entities on the map (e.g., Facebook Places [1]).

Still, humans can make mistakes. Even for humans, questions for some specific pairs of records are inherently difficult to answer correctly. Moreover, in large-scale crowdsourcing platforms, a non-negligible portion of workers are spammers, or provide low quality answers because they do not pay full attention to the tasks.

In this paper, we study the problem of entity resolution where evidence is collected on-demand from possibly erroneous humans. That is, given a set of records, we find which questions between pairs of records will reveal "the most" about the underlying entities. Our solution also takes into account evidence from traditional record similarity functions. (For instance, a computer can compare the author, title and venue fields of records representing two articles to determine with some confidence if the records represent the same article.)

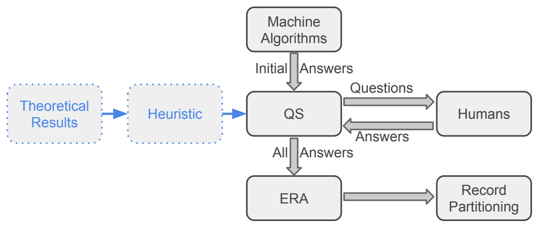


Fig. 1. Overview or our approach

A brute force approach would be to consider every possible pair of records in the data set, and to ask a human if that pair represents the same real world entity. Since humans can make mistakes, we would further repeat the same question to multiple humans until we were confident we had the correct answer for that question. While this scheme would produce the most evidence, it is clearly infeasible when more than a handful of records are involved: Humans take time to answer questions and usually have to be paid. Instead, our approach (Fig. 1) is to first generate as much computer evidence as possible, which we assume is a fast and inexpensive process (if not very accurate). Then, based on the evidence on hand, we select one or more pairs of records, and ask humans for their answers. After incorporating the new evidence, we repeat the process, ending when we are "confident enough" that we can partition the records into entities; or when we exhausted our budget. At that point, we apply an entity resolution algorithm (ERA): the ERA takes the probabilistic evidence and partitions the records, where each cluster represents the records thought to represent one real world entity. We base our Question Selection (QS) on a heuristic driven by a number of interesting theoretical results that capture the usefulness of each question. The details are presented in the rest of this paper.

The main difference of our approach from others is the way we handle human errors. Some papers (like [2], [3], [1]) assume that humans make no mistakes. In such a scenario, the only uncertainty comes from the computer generated similarities, so it becomes easier to reason about what questions to ask. Most techniques [4], [5], [6] for dealing with human errors, end up in repeating a question to n humans, and making n large enough so that the "majority" answer is effectively error-free. As we will see in our experiments, this approach works in some cases, but in others may be wasteful. Another paper [7] does take into account human errors, but selects record pairs to resolve next at random.

Our solution, on the other hand, determines questions to ask based on the evidence collected (both machine and human evidence), in order to increase the probability of the Maximum Likelihood(ML) clustering. In a sense, our approach finds the best possible questions to ask, independent of the ERA. Our scheme is not constrained to ask fixed blocks of n questions for each pair of records, and is free to allocate questions as it best sees fit.

In addition to our question selection strategy, in this paper we also present an interesting ERA algorithm that works especially well with probabilistic evidence. We call this algorithm SCC (Spectral-Connected Components). A traditional ERA would first apply a probability threshold to convert the evidence into a deterministic graph. For example, nodes x and y are connected in the graph if the probability that they are the same entity is higher than say 0.9. Then the traditional ERA would apply some global clustering algorithm to partition the graph. On the other hand, SCC forms clusters by examining the full evidence. For instance, to decide it two clusters should merge into one, it considers all questions that involve records in the two clusters in question. As we will see, SCC yields high quality results, not just when used with our own question selection strategy, but also when used with other strategies.

In summary, we make the following contributions:

- We provide a formal definition for the question selection problem, under the assumption that human answers may contain errors, in Section II-C.
- We provide theoretical results that identify "beneficial" questions with a reasonable computational cost, in Section III.
- We propose a lightweight algorithm that applies our theoretical results to select questions, in Section IV.
- We present our SCC clustering algorithm for partitioning records based on probabilistic evidence, in Section V.
- We compare our question selection algorithm to two algorithms [2], [3] proposed in related work, using datasets of images and AMT [8] workers, and we identify in which cases each algorithm is better, in Section VI.

A. *Running Example*

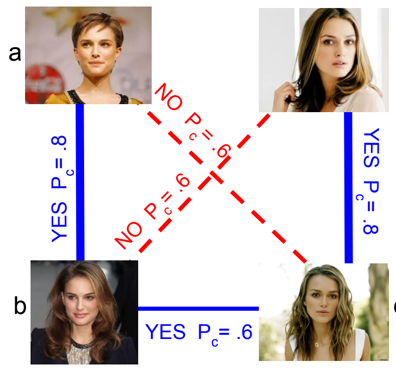


Fig. 2. Entity resolution in a set of four images

Consider the dataset of 4 records in Fig. 2. The same person appears in images a and b and another person appears in images c and d. We are provided with 5 answers: a) YES from a human for the pair (a, b), b) YES from a human for the pair (c, d), c) YES from an image processing algorithm for the pair (b, d), d) NO from an image processing algorithm for the pair (a, d), and e) NO from an image processing algorithm for the pair (b, c). In this example, say humans make mistakes 80% of the time. Therefore, along with each human answer we have a probability of being correct, denoted by p_c^H, equal to 0.8. In addition, say the image processing algorithm generates its three answers with confidence 0.6. Thus, we associate this value, denoted by p_c^M, with its answers. Our objective is to select which additional questions to send to humans.

II. Preliminaries

A. *Model*

Our model assumes that between pairs of records we have YES/NO answers, from machines or humans. A YES answer is correct, if the two records actually refer to the same entity, while a NO answer is correct, if the two records refer to different entities. Note the subtle difference between this model and the model of [3]. To illustrate, consider three records a, b and c that refer to a single entity. In [3], a human, who is always correct, can answer NO to the pair a-b, as long as he (or another human) answers YES to b-c and to c-a (so that the correct entity can be discovered through transitive closure). In our case, the correct answer to all three questions is YES because the records are part of the same entity, independent of how entities are discovered by the ERA.

Along with each answer, there is a probability of the answer being correct. For human answers, we assume that there is a single probability for any record-pair answer provided by a human. We call this probability p_c^H. In a specific crowdsourcing platform, p_c^H can be the expected accuracy of humans for a specific type of dataset.

In practice, some record pairs may be harder than others for humans to answer. However, we make the single p_c^H error rate assumption to simplify the question selection reasoning. In our experimental evaluation we show that in data sets where the assumption does not hold, our approach still quickly converges to high quality results.

We assume that humans involved in the process are not malicious, i.e., the worst a human can do is guess at random. Thus, p_c^H is never lower than 0.5. For machine answers, there can be a different probability for each record-pair answer. Since most machine algorithms generate a similarity value between two records, this similarity value needs to be translated to a probability, P, of the two records referring to the same entity. If P is lower than 0.5, then we need to generate a NO answer with probability of being correct $1 - P$, otherwise, we will generate a YES answer with probability P.

In the machine answer case, the similarity-to-probability process must take into account the amount of information used for generating the similarity value. For example, consider two records, a and b, with 3 fields, full-name, phone-number, and country. If a and b have identical values in all 3 fields, they will have a max similarity value of 1.0. On the other hand, two records, c and d, missing values on full-name and phone-number, and having the same country value will also have similarity 1.0. However, the probability of referring to the same entity for c and d should be significantly lower than a and b. Moreover, if two other records, e and f, were missing values in full-name and country and had the same phone-number value, they should receive a higher probability than c and d, because the phone number is more indicative than country. Note also that how indicative some fields are, depends on the specific dataset. For example, fields brand and type in a dataset containing only Apple laptops are not indicative at all. In Section VI-A3, we will examine how a low quality similarity-to-probability translation can affect our approach.

B. *Maximum Likelihood Clustering*

In this section, we define the Maximum Likelihood (ML) clustering, which will be a main building block in the definition of the "best" next question, in the next section.

Let us denote the answers which already appear in our dataset as evidence E. In Fig. 2, E consists of five answers.

First, we will examine the most probable clustering of records in Fig. 2, based on E, such that all records of each formed cluster refer to one entity. An example of such a clustering appears in Fig. 3(a), where all the records are clustered into a single entity.

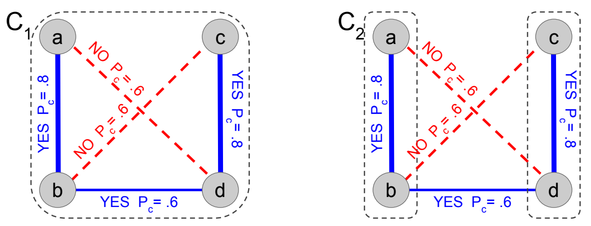
The probability of each clustering C_k given the evidence E, is $P(C_k|E)$, and we want to find

$$\max_k P(C_k|E) = \max_k \frac{P(C_k, E)}{P(E)} = \max_k \frac{P(E|C_k)P(C_k)}{P(E)} \tag{1}$$

Therefore, we want to find the clustering C_k that maximizes $P(E|C_k)P(C_k)$.

We assume that each clustering has the same a priori probability, $P(C_k)$. In other words, we assume that we don't have any other prior knowledge about the dataset, except for the evidence E. Other prior knowledge that could have been available, is statistics regarding our dataset, e.g., distribution for the number of records referring to the same entity. In Section VIII we discuss some possible extensions to our model, for future work.

Since $P(C_k)$ is the same for any clustering C_k, we will simply have to find the clustering maximizing $P(E|C_k)$, i.e., the ML clustering. For example, consider the clustering $C_1 = (\{a, b, c, d\})$ depicted in Fig. 3(a). $P(E|C_1) = 0.8^2 * 0.6 * (1 - 0.6)^2$, i.e., answers about the pairs, (a, b), (c, d) and (b, d) are proven to be correct, while answers for the pairs (a, d) and (b, c) are proven to be false, if C_1 is the "true" clustering.



(a) C_1 (b) C_2

Fig. 3. Clusterings C_1 and C_2

A clustering with a higher likelihood $P(E|C_k)$ is clustering $C_2 = (\{a, b\}, \{c, d\})$, depicted in Fig. 3(b). $P(E|C_2) = 0.8^2 * 0.6^2 * (1 - 0.6)$, i.e., answers about the pairs, (a, b), (c, d), (a, d) and (b, c) are proven to be correct, while the answer for (b, d) is proven to be false, if C_2 is the "true" clustering. Note that C_2 is the ML clustering for the five answers we are given, i.e., no other clustering can do better.

For the rest of the paper, we will denote the probability $P(E|C_{ML})$ of the ML clustering, C_{ML}, by $\mathcal{ML}(E)$, i.e.,

$$\mathcal{ML}(E) = \max_k P(E|C_k) \tag{2}$$

Finding the ML clustering is NP-hard, by reduction from correlation clustering [9]. In correlation clustering the objective is to find a clustering that minimizes the number of "disagreements". For instance, in Fig. 3(b), the YES between records b and d forms a disagreement for clustering C_2. In this paper, however, we are interested in addressing a different problem; finding the questions to ask next. In the next section, we provide a formal definition for the simple case of this problem, where we are interested in finding the next single question to ask next.

C. Next Single Question

In this section, we give a definition for the "best" single question to ask next. Using this definition, we unfold a number of theoretical results, in Section III, that form the basis of our algorithms, described in Section IV.

Let us examine which would be the "best" single question to ask next. For a question q_{ij} between records i and j, we may get back a NO, denoted by n_{ij}, or a YES, denoted by y_{ij}. We define the *Expected Probability* (EP) of the ML clustering (technically we start from the Maximum A Posteriori(MAP) clustering), over the cases of a YES and a NO, for a question q_{ij} as:

$$\begin{aligned} EP_{ij} &= P(y_{ij}|E) \max_k P(C_k|E \wedge y_{ij}) \\ &+ P(n_{ij}|E) \max_k P(C_k|E \wedge n_{ij}) \end{aligned} \tag{3}$$

In case of a YES, we have:

$$\begin{aligned} P(y_{ij}|E) \max_k P(C_k|E \wedge y_{ij}) &= \\ \tfrac{P(E \wedge y_{ij})}{P(E)} \max_k \tfrac{P(E \wedge y_{ij}|C_k)P(C_k)}{P(E \wedge y_{ij})} &= \\ \tfrac{1}{P(E)} \max_k P(E \wedge y_{ij}|C_k)P(C_k) & \end{aligned} \tag{4}$$

Likewise, in case of a NO we have:

$$\begin{aligned} P(n_{ij}|E) \max_k P(C_k|E \wedge n_{ij}) &= \\ \tfrac{1}{P(E)} \max_k P(E \wedge n_{ij}|C_k)P(C_k) & \end{aligned} \tag{5}$$

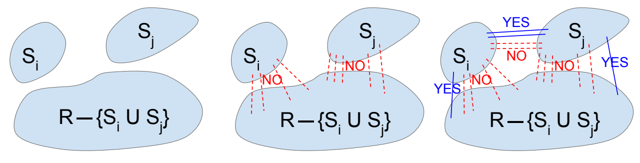
Since, we assume equal a priori clustering probabilities, i.e., $\forall k$, $P(C_k) = P_C$, we have:

$$\begin{aligned} EP_{ij} &= \frac{P_C}{P(E)} \left(\max_k P(E \wedge y_{ij}|C_k) + \max_k P(E \wedge n_{ij}|C_k) \right) \\ &= \frac{P_C}{P(E)} \left(\mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) \right) \end{aligned} \tag{6}$$

Therefore, the "best" single question to ask next, is the question q_{ij} that maximizes:

$$\alpha(ij) = \mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) \tag{7}$$

$\alpha(ij)$ is the probability, in expectation, of the MAP clustering for the new evidence (including the answer for q_{ij}), divided by the constant-for-all-questions quantity of $\frac{P_C}{P(E)}$.



(a) COROLLARY 1 (b) COROLLARY 2 (c) COROLLARY 3

Fig. 4. The cases where COROLLARIES 1, 2, and 3 apply.

III. Theoretical Results

In this section, we discuss our main theoretical results: THEOREM 1 and its three COROLLARIES. The results from the corollaries are directly applied by our algorithm in Section IV. We first provide an overview before discussing the details. Each corollary applies to a particular scenario, and provides suggestions for good questions to ask in each case. COROLLARY 1 applies to the scenario of Fig. 4(a), where we can identify three sets of records, S_i, S_j, and $R - \{S_i \cup S_j\}$, and there are no answers between pairs from different sets. (R is the set of all records.) COROLLARY 1 states that in this case, any question between two records from different sets is one of "best" questions to ask. COROLLARY 2 applies to a more general case, depicted in Fig. 4(b), where there are NO answers between records of S_i(or S_j) and $R-\{S_i \cup S_j\}$. In this scenario, a question between an S_i record and an S_j record is one of the "best" questions to ask. Extending even further the scenario of COROLLARY 2 we add *a)* YES answers between S_i(or S_j) and $R-\{S_i \cup S_j\}$ and *b)* YES/NO answers between S_i and S_j, in Fig. 4(c). In this case, COROLLARY 3 provides guarantees on how "good" or "bad" a question between S_i and S_j can be, based on the "weights" of the added answers.

We start with some preliminary results and definitions that will simplify the statements and proofs of THEOREM 1 and COROLLARIES 1, 2, and 3. While in our definition of the "best" question to ask next, the NP-hard problem of ML clustering is involved, our main results, in COROLLARIES 1, 2, and 3, do not assume knowledge of the ML clustering.

LEMMA 1. $\forall\ q_{ij},\ \alpha(ij) \in [\mathcal{ML}(E), 2p_c^H\mathcal{ML}(E)]$

PROOF. First, note that since human participants are not malicious, and $p_c^H \geq 0.5$, it follows that $2p_c^H\mathcal{ML}(E) \geq \mathcal{ML}(E)$. Consider an arbitrary question q_{ij}. Next, we examine the best and worst case scenarios:

1) *YES answer for* q_{ij}
   a) *Best case scenario:* There is an ML clustering, C_{ML} for E, that has records i and j in the same cluster. In that case the YES answer we got back is considered correct for C_{ML}. Therefore,

$$\mathcal{ML}(E \wedge y_{ij}) = p_c^H\mathcal{ML}(E)$$

   b) *Worst case scenario:* Any ML clustering for E has records i and j in two different clusters. For $E \wedge y_{ij}$, there are two cases for the ML clustering:
      i) *Any ML clustering for $E \wedge y_{ij}$ has i and j in two different clusters.* In this case, the new YES answer is considered wrong and:

$$\mathcal{ML}(E \wedge y_{ij}) = (1-p_c^H)\mathcal{ML}(E)$$

      ii) *There is an ML clustering for $E \wedge y_{ij}$ with i and j in the same cluster.* Consider C, an ML clustering for E. $P(E \wedge y_{ij}|C) = (1-p_c^H)\mathcal{ML}(E)$, since any ML clustering for E has records i and j in two different clusters. If C is an ML clustering for $E \wedge y_{ij}$ then,

$$\mathcal{ML}(E \wedge y_{ij}) = (1-p_c^H)\mathcal{ML}(E)$$

      Otherwise,

$$\mathcal{ML}(E \wedge y_{ij}) > (1-p_c^H)\mathcal{ML}(E)$$

2) *NO answer for* q_{ij}
   a) *Best case scenario:* There is an ML clustering, C_{ML} for E, that has records i and j in two different clusters. Thus, the NO answer is considered correct for C_{ML} and

$$\mathcal{ML}(E \wedge n_{ij}) = p_c^H\mathcal{ML}(E)$$

   b) *Worst case scenario:* Any ML clustering for E has records i and j in the same cluster. For $E \wedge n_{ij}$, there are two cases for the ML clustering:
      i) *Any ML clustering for $E \wedge n_{ij}$ has i and j in the same cluster.* In this case, the new NO answer is considered wrong and:

$$\mathcal{ML}(E \wedge n_{ij}) = (1-p_c^H)\mathcal{ML}(E)$$

      ii) *There is an ML clustering for $E \wedge n_{ij}$ with i and j in two different clusters.* Consider C, an ML clustering for E. $P(E \wedge n_{ij}|C) = (1-p_c^H)\mathcal{ML}(E)$, since any ML clustering for E has records i and j in the same cluster. If C is an ML clustering for $E \wedge n_{ij}$ then,

$$\mathcal{ML}(E \wedge n_{ij}) = (1-p_c^H)\mathcal{ML}(E)$$

      Otherwise,

$$\mathcal{ML}(E \wedge n_{ij}) > (1-p_c^H)\mathcal{ML}(E)$$

Now we examine the overall worst and best case scenarios:

- *Overall best case scenario:* The upper bound for $\alpha(ij)$ is achieved when the best case scenarios (1a) and (2a) happen. This is possible, when there are at least two ML clusterings for E, one having records i and j in the same cluster and one having the two records in different clusters. In that case,

$$\alpha(ij) = \mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) = 2p_c^H\mathcal{ML}(E)$$

- *Overall worst case scenario:* For the lower bound, note that when the best case scenario (1a) does not happen, then the best case scenario (2a) must hold. If (1a) does not hold then there will be an ML clustering for E with records i and j in different clusters, and, therefore, case (2a) will hold. In a similar fashion, if case (2a) does not hold, (1a) must hold. In the worst case, either cases (1b) and (2a) hold, or (1a) and (2b) hold. Hence,

$$\alpha(ij) = \mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) = \\ (p_c^H + 1 - p_c^H)\mathcal{ML}(E) = \mathcal{ML}(E)$$

□

An example where the upper bound is achieved for $\alpha(ij)$ is given in Fig. 5. Initial evidence E consists of four answers and there are two ML clusterings for E, $C_1 = (\{a, b\}, \{c, d\})$ and $C_2 = (\{a, b, c, d\})$. Consider images a and c. Both of the best case scenarios, (1a) and (2a), take place, i.e., in C_1, a and c are in different clusters, while in C_2, a and c are in the same cluster. Thus, question q_{ac} achieves the upper bound, $2p_c^H\mathcal{ML}(E)$. This is also the case for questions q_{bd}, q_{bc}, and q_{ad}.

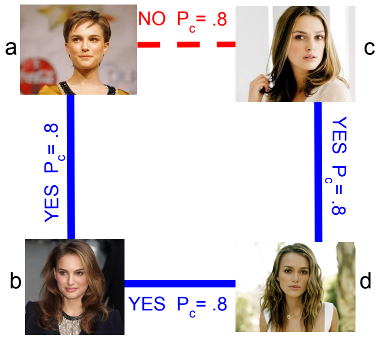


Fig. 5. Upper α bound achieved by q_{ac},q_{bd}, q_{bc}, and q_{ad}.

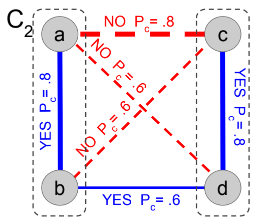
Intuitively, a question on the lower bound, i.e., with $\alpha = \mathcal{ML}(E)$, does not increase our confidence for the ML clustering. On the other hand, any question with $\alpha > \mathcal{ML}(E)$, i.e., strictly higher than the lower bound, will increase, even slightly, our confidence in the ML clustering; whether the question is answered YES or NO. We will call a question with an α strictly higher than the lower bound, ***beneficial***. Now, the most *beneficial* questions that achieve the upper bound, $2p_c^H\mathcal{ML}(E)$, are called ***optimal***, in the sense that no other question can do better.

DEFINITION 1. ***beneficial question*** **:** *A question with an* α *strictly higher than* $\mathcal{ML}(E)$ *, is a* beneficial question.

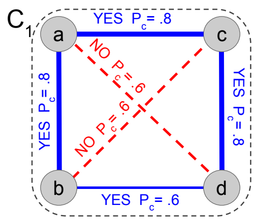
DEFINITION 2. ***optimal question*** **:** *A question with an* α *equal to* $2p_c^H\mathcal{ML}(E)$ *, is an* optimal question.

For instance, q_{ac} is a *beneficial* question in our running example of Fig. 2. A NO gives C_2 as the ML clustering, as shown in Fig. 6(a), with

$$\mathcal{ML}(E \wedge n_{ac}) = p_c^H P(E|C_2) = p_c^H\mathcal{ML}(E)$$



(a) C_2 after a NO for q_{ac}



(b) C_1 after a YES for q_{ac}

Fig. 6. Clusterings C_2 and C_1 after q_{ac} is answered

A YES gives C_1 as the ML clustering, as shown in Fig. 6(b), with

$$\mathcal{ML}(E \wedge y_{ac}) = p_c^H P(E|C_1) = p_c^H \frac{0.6 * (1-0.6)^2}{0.6^2 * (1-0.6)}\mathcal{ML}(E)$$

since $P(E|C_1) = 0.8^2 * 0.6 * (1-0.6)^2$ and $\mathcal{ML}(E) = 0.8^2 * 0.6^2 * (1-0.6)$. Thus,

$$\alpha(ac) = p_c^H(1 + \frac{0.6 * (1-0.6)^2}{0.6^2 * (1-0.6)})\mathcal{ML}(E) \simeq 1.33\mathcal{ML}(E)$$

On the other hand, a question which is not *beneficial* is question q_{ab}, because

$$\begin{aligned}\alpha(ab) &= \mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) \\ &= p_c^H\mathcal{ML}(E) + (1-p_c^H)\mathcal{ML}(E) = \mathcal{ML}(E)\end{aligned}$$

In this example, the pair of records (a, b) seems to refer to one entity and the pair (c, d) also seems to refer to one entity. Still, it is not clear if (a, b) refers to a different entity than (c, d), or if all four records refer to one single entity. Thus, asking q_{ac} is more "useful" than asking q_{ab}. This notion of "uncertainty" is formally defined by λ***–balance***.

λ*–balance* expresses how "uncertain" we are about two clusters of an ML clustering. Should they be merged into one single entity or should they remain two separate clusters/entities? The higher the value of λ, the higher the uncertainty. When λ reaches its maximum value, 1.0, we practically have no evidence for keeping the two clusters as two separate entities. On the contrary, a λ value of 0.0 indicates that the two clusters should definitely be kept as two separate clusters/entities. Below, we formally define λ*–balance* and we prove that $\lambda \in [0.0, 1.0]$.

DEFINITION 3. λ***–balance*****:** *Consider an ML clustering with two clusters c_1 and c_2, as given in Fig. 7(a). Between c_1 and c_2, we have a set of YES answers, Y, and a set of NO answers, N, from humans and machines; for each answer of Y and N one record is in c_1 and the other in c_2. An answer $a \in \{Y \cup N\}$ has a probability of being correct $p_c(a)$. We denote:*

$$P_Y = \prod_{a \in Y} p_c(a), P'_Y = \prod_{a \in Y}(1 - p_c(a)) \tag{8}$$

$$P_N = \prod_{a \in N} p_c(a), P'_N = \prod_{a \in N}(1 - p_c(a)) \tag{9}$$

and we say that there is a λ–balance *between c_1 and c_2, for $\lambda = \frac{P'_N * P_Y}{P_N * P'_Y}$.*

LEMMA 2. *In a* λ–balance *between two clusters of an ML clustering,* $\lambda \in [0.0, 1.0]$

PROOF. Let us denote the ML clustering by C_A and the two clusters as c_1 and c_2. If the overall evidence is the set of answers E, we have:

$$\mathcal{ML}(E) = P(E - \{Y \cup N\}|C_A) * P_N * P'_Y$$

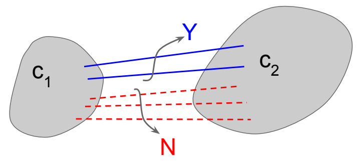
Now, consider a clustering $C_{A'}$ that is identical to C_A, with the only difference of having clusters c_1 and c_2 merged into one single cluster. For $C_{A'}$ we have $P(E - \{Y \cup N\}|C_{A'}) = P(E - \{Y \cup N\}|C_A)$, and, thereafter,

$$P(E|C_{A'}) = P(E - \{Y \cup N\}|C_A) * P'_N * P_Y$$

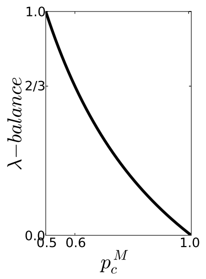
Because C_A is an ML clustering, $\mathcal{ML}(E) \geq P(E|C_{A'})$. Hence, $P_N * P'_Y \geq P'_N * P_Y$, and,

$$\lambda = \frac{P'_N * P_Y}{P_N * P'_Y} \leq 1.0$$

In addition, $\lambda \geq 0.0$, because probability is always non-negative.



(a) λ–*balance* definition for two clusters



(b) λ–*balance* as p_c^M goes from 0.5 to 1.0, in the running example

Fig. 7. λ–*balance* description

□

In the ML clustering of our running example, in Fig. 3(b), between the two clusters $\{a, b\}$, $\{c, d\}$, we have $N = \{n_{ad}, n_{bc}\}$, $Y = \{y_{bd}\}$, $P_N = 0.6^2$, $P'_N = (1 - 0.6)^2$, $P_Y = 0.6$ and $P'_Y = (1-0.6)$. A λ–*balance* exists between the two clusters, for $\lambda = \frac{(1-0.6)^2 * 0.6}{0.6^2 * (1-0.6)} = \frac{2}{3}$. In case the probability of a correct answer from a machine, p_c^M, for the three machine answers, n_{ad}, n_{bc}, and y_{bd}, was lower than 0.6, then the λ–*balance* between $\{a, b\}$ and $\{c, d\}$, would be higher than $\frac{2}{3}$. On the other hand, if p_c^M was higher than 0.6, λ would be lower than $\frac{2}{3}$. In Fig. 7(b), we plot the λ–*balance* between $\{a, b\}$ and $\{c, d\}$, as p_c^M goes from 0.5 to 1.0.

The following theorem connects the notions of λ–*balance* and the α value of a new question. While the theorem assumes knowledge of the ML clustering, we present three corollaries that do not assume such knowledge. These corollaries identify practical cases for finding *beneficial* questions, since finding the ML clustering is NP-hard. In Section IV, we will present an algorithm that makes use of the results from the corollaries.

THEOREM 1. *If an ML clustering contains two clusters with a* λ–balance *between them, for* $\lambda > \frac{1-p_c^H}{p_c^H}$, *any question between a record* i *from the first cluster and a record* j *from the second cluster, is a* beneficial *question. In addition,*

$$\alpha(ij) \geq (1 + \lambda) p_c^H \mathcal{ML}(E)$$

PROOF. Let C_{ML} be the ML clustering, c_1 and c_2 the two clusters and q_{ij} the arbitrary question we ask, where i belongs to c_1 and j to c_2.

1) *NO answer for* q_{ij}:

$$\mathcal{ML}(E \wedge n_{ij}) = p_c^H \mathcal{ML}(E)$$

since the ML clustering for E is also an ML clustering for $E \wedge n_{ij}$.

2) *YES answer for* q_{ij}: Consider the clustering C_{mrg} which is the exact same clustering with C_{ML}, except that we merge c_1 and c_2 into one cluster. The likelihood of C_{mrg} for our initial evidence E, is $\lambda \mathcal{ML}(E)$. For $E \wedge y_{ij}$ the likelihood of C_{mrg} becomes $p_c^H \lambda \mathcal{ML}(E)$. Thus, the ML clustering for $E \wedge y_{ij}$ has a likelihood of at least $p_c^H \lambda \mathcal{ML}(E)$, and

$$\mathcal{ML}(E \wedge y_{ij}) \geq p_c^H \lambda \mathcal{ML}(E)$$

For question q_{ij},

$$\alpha(ij) = \mathcal{ML}(E \wedge n_{ij}) + \mathcal{ML}(E \wedge y_{ij}) \geq (1 + \lambda) p_c^H \mathcal{ML}(E)$$

Moreover, since $\lambda > \frac{1-p_c^H}{p_c^H}$, $\alpha(ij) > (1 + \frac{1-p_c^H}{p_c^H}) p_c^H \mathcal{ML}(E) = \mathcal{ML}(E)$. Therefore, q_{ij} is a *beneficial* question.

□

An application of THEOREM 1 appears in the running example, in Fig. 3(b), where there is a λ–*balance* between the two clusters, $\{a, b\}$, $\{c, d\}$, of the ML clustering, for $\lambda = \frac{(1-0.6)^2 * 0.6}{0.6^2 * (1-0.6)}$. Questions q_{ac}, q_{ad}, q_{bc}, q_{bd} are *beneficial* with an α of, at least, $(1 + \lambda) 0.8 \mathcal{ML}(E) \approx 1.33 \mathcal{ML}(E)$; as we discussed before, α is exactly that for these 4 questions.

The three following corollaries are based on the notion of λ–*balance*, and define cases where we can "easily" find *optimal* or *beneficial* questions. During the statements and proofs of the three corollaries, we use the following notation:

- $\mathcal{G}$: The graph formed by the YES and NO answers from machines and humans collected so far. The nodes of the graph are the records of the dataset and an edge between two records, consists of an answer from a human or machine for the two records.
- $\mathcal{G}_Y$: $\mathcal{G}$ without the NO edges.

COROLLARY 1. *If* $\mathcal{G}$ *has two or more connected components, a question between any record* i *from one connected component and any record* j *from another connected component, is an* optimal *question.*

PROOF. Consider the two connected components, S_i and S_j, and pick arbitrarily a record i from the first and a record j from the second. This situation is depicted in Fig. 4(a); the "Rest of the Graph" can be the empty set. Moreover, consider an ML clustering C_{ML}, for the evidence E. There are two cases:

1) *In* C_{ML}, i *and* j *belong to the same cluster.* Let us denote this cluster by c and remove some of the elements from it, in order to construct two new clusters. Specifically, we construct clusters $c'_i \equiv c \cap S_i$ and $c'_j \equiv c \cap S_j$. Moreover, by removing elements from c we construct a third new cluster, $c' \equiv c - \{c'_i \cup c'_j\}$. Since S_i and S_j are two separate connected components, there are no answers between c'_i and c'_j, between c' and c'_i, and between c' and c'_j. Therefore, the new clustering does not "violate" any answer and it is also an ML clustering for E. In addition, in the new clustering there is a 1.0–*balance* between clusters c'_i and c'_j, since no question has been asked between their elements. Based on THEOREM 1, $\alpha(ij) \geq (1 + 1) p_c^H \mathcal{ML}(E)$ and, therefore, q_{ij} is an *optimal* question.
2) *In* C_{ML}, i *and* j *belong to different clusters.* Let us denote these clusters by c_i, for record i and c_j for record j. We apply the same construction process as in case (1), and we form two new clusters $c'_i \equiv c_i \cap S_i$ and $c'_j \equiv c_j \cap S_j$. Again, in the new clustering there is a

1.0–*balance* between clusters c'_i and c'_j, and, based on THEOREM 1, q_{ij} is an *optimal* question.

□

COROLLARY 1 can be generalized to the case of COROLLARY 2, illustrated in Fig. 4(b).

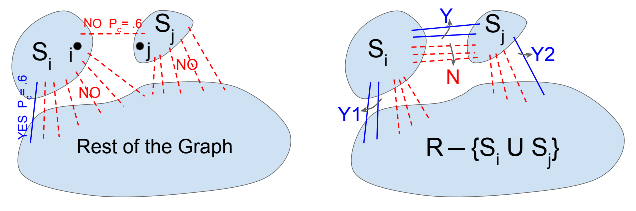
COROLLARY 2. *Consider two connected components of $\mathcal{G}_Y$ without any NO answers between a record from the first and a record from the second component. If such pair of components exist, a question between any record of the first component and any record from the second one, is an* optimal *question.*

PROOF. Consider two connected components, S_i and S_j, for which COROLLARY 2 applies, and two arbitrarily picked records i and j from the two components. The case is depicted in Fig. 4(b). We can follow the exact same construction process as in the proof of COROLLARY 1 and get an ML clustering with two new clusters c'_i and c'_j. Again, between c'_i and c'_j there is a 1.0–*balance* and q_{ij} is an *optimal* question.

□

In order to find an optimal question, COROLLARY 2 suggests that we can *1)* remove all the NO answers from $\mathcal{G}$, *2)*find the connected components, and *3)* try all the pairs of connected components until we find two components without any NO answers between a record of the first component and a record of the second one. If we find such a pair of components, we can arbitrarily select one record from the first component and one record from the second component and issue a question for this record pair. In Section IV we discuss in detail the complete algorithm that uses the corollaries presented here.

Now, let us consider a more general case, shown in Fig. 8(a). The situation is the same with the one depicted in Fig. 4(b), except for *a)*a YES answer with probability of being correct $p_c = 0.6$, between the S_i and the "Rest of the Graph" and *b)* a NO answer between S_i and S_j, again with $p_c = 0.6$. How "bad" can a question between arbitrarily picked records i from S_i and j from S_j be, in this case? Or, in other words, is there a lower bound for $\alpha(ij)$? Such a guarantee would help us find *beneficial* questions. As COROLLARY 3 states, q_{ij} is a *beneficial* question, with $\alpha(ij) \geq (1+\frac{0.4^2}{0.6^2})p_c^H\mathcal{ML}(E)$. Next, we give the general definition for the ratio $\frac{0.4^2}{0.6^2}$ resulting from answers *a)* and *b)*, in this example.



(a) Motivation example (b) General case where COROLLARY 3 applies

Fig. 8. COROLLARY 3 description

DEFINITION 4. ***ρ–ratio**: Consider two disjoint subsets of records, S_i and S_j, of the overall set of records R, as they appear in Fig. 8(b). We denote* a) *the YES answers between S_i and $R-\{S_i \cup S_j\}$ by $Y1$,* b) *the YES answers between S_j and $R-\{S_i \cup S_j\}$ as $Y2$,* c) *the NO answers between S_i and S_j as N, and* d) *the YES answers between S_i and S_j as Y. The probability of an answer $a \in \{Y1 \cup Y2 \cup N \cup Y\}$ being correct, is $p_c(a)$. In addition, we denote 1) $P_{Y1} = \prod_{a\in Y1} p_c(a)$, 2)$P_{Y2} = \prod_{a\in Y2} p_c(a)$, 3)$P_N = \prod_{a\in N} p_c(a)$, 4)$P_Y = \prod_{a\in Y} p_c(a)$, 5) $P'_{Y1} = \prod_{a\in Y1}(1-p_c(a))$, 6) $P'_{Y2} = \prod_{a\in Y2}(1-p_c(a))$, 7)$P'_N = \prod_{a\in N}(1-p_c(a))$, and 8)$P'_Y = \prod_{a\in Y}(1-p_c(a))$.*

*We call ρ–*ratio*, the ratio:*

$$\frac{P'_{Y1} * P'_{Y2}}{P_{Y1} * P_{Y2}} * \min\{\frac{P'_N}{P_N}, \frac{P'_Y}{P_Y}\} \tag{10}$$

COROLLARY 3. *When between two disjoint subsets of records, S_i and S_j, there is a ρ–*ratio *with*

$$\rho > \frac{(1-p_c^H)}{p_c^H} \tag{11}$$

a question between an arbitrary record i of S_i and an arbitrary record j of S_j is a beneficial *question.*

Moreover,

$$\alpha(ij) \geq (1+\rho)\, p_c^H \mathcal{ML}(E) \tag{12}$$

PROOF. Consider two arbitrarily picked records i and j, from S_i and S_j, as they appear in Fig. 8(b). Let c_i be the cluster of i and c_j the cluster of j, in an ML clustering, C_{ML}. Next, we are going to describe the worst case scenario when (1) clusters c_i and c_j are two different clusters and (2) clusters c_i and c_j are the same cluster.

1) *c_i and c_j are two different clusters*
   a) *NO answer for q_{ij}:* The ML clustering for $E \wedge n_{ij}$ is C_{ML}, with likelihood equal to $p_c^H\mathcal{ML}(E)$.
   b) *YES answer for q_{ij}:* In this case, the likelihood of the ML clustering for $E \wedge y_{ij}$ may be a lot less than $p_c^H\mathcal{ML}(E)$. We can get a lower bound on $\mathcal{ML}(E \wedge y_{ij})$, by considering the following clustering. Remove from c_i the subset $c_i \cap S_i$, remove from c_j the subset $c_j \cap S_j$, and create a new cluster $\{c_i \cap S_i\} \cup \{c_j \cap S_j\}$. The likelihood of the constructed clustering for $E \wedge y_{ij}$ is, at least
   $$\frac{P'_{Y1} * P'_{Y2} * P'_N}{P_{Y1} * P_{Y2} * P_N} p_c^H \mathcal{ML}(E)$$
   because we can not "violate" any additional answers except for those in $\{Y1 \cup Y2 \cup N\}$.
   Hence,
   $$\mathcal{ML}(E \wedge y_{ij}) \geq \frac{P'_{Y1} * P'_{Y2} * P'_N}{P_{Y1} * P_{Y2} * P_N} p_c^H \mathcal{ML}(E)$$
2) *c_i and c_j are the same cluster c*
   a) *YES answer for q_{ij}:* The ML clustering for $E \wedge y_{ij}$ is C_{ML}, with likelihood equal to $p_c^H\mathcal{ML}(E)$.
   b) *NO answer for q_{ij}:* We follow a similar construction process as in (1b) to provide a lower bound on $\mathcal{ML}(E \wedge n_{ij})$. We consider the following clustering. Remove from c the subset $c \cap S_i$, remove from c the subset $c \cap S_j$, and create two new

clusters $\{c \cap S_i\}$ and $\{c \cap S_j\}$. The likelihood of the constructed clustering for $E \wedge n_{ij}$ is, at least

$$\frac{P'_{Y1} * P'_{Y2} * P'_{Y}}{P_{Y1} * P_{Y2} * P_{Y}} p_c^H \mathcal{ML}(E)$$

because we can not "violate" any additional answers except for those in $\{Y1 \cup Y2 \cup Y\}$.

Hence,

$$\mathcal{ML}(E \wedge y_{ij}) \geq \frac{P'_{Y1} * P'_{Y2} * P'_{Y}}{P_{Y1} * P_{Y2} * P_{Y}} p_c^H \mathcal{ML}(E)$$

By combining cases (1) and (2), we get the overall worst case scenario, where we have:

$$\alpha(ij) = \mathcal{ML}(E \wedge y_{ij}) + \mathcal{ML}(E \wedge n_{ij}) \geq \left(1 + \frac{P'_{Y1} * P'_{Y2}}{P_{Y1} * P_{Y2}} * \min\{\frac{P'_N}{P_N}, \frac{P'_Y}{P_Y}\}\right) p_c^H \mathcal{ML}(E)$$

Since

$$\rho = \frac{P'_{Y1} * P'_{Y2}}{P_{Y1} * P_{Y2}} * \min\{\frac{P'_N}{P_N}, \frac{P'_Y}{P_Y}\} > \frac{(1 - p_c^H)}{p_c^H}$$

we have:

$$\begin{aligned} \alpha(ij) &\geq (1 + \rho) p_c^H \mathcal{ML}(E) \\ &> (1 + \frac{(1 - p_c^H)}{p_c^H}) p_c^H \mathcal{ML}(E) = \mathcal{ML}(E) \end{aligned}$$

Therefore, q_{ij} is a *beneficial* question.

□

IV. Question Selection Algorithm

As COROLLARY 3 points out, we can find a *beneficial* question, by detecting two disjoint sets of records that *a)* are not "strongly" connected with YES answers with the rest of the dataset and *b)* between the two sets there is neither "strong" YES evidence, nor "strong" NO evidence. The notion of "strong" is quantified by the *ρ–ratio*. In other words, COROLLARY 3 suggests a way to detect "lack"-of-evidence inside the dataset.

While the heuristic we present in this section does not always find an *optimal* or even a *beneficial* question (in the formal sense), it detects "lack"-of-evidence and picks questions that strongly enhance the pre-existing evidence. In the experimental evaluation, we show that as we keep asking questions that our heuristic selects, we rapidly converge to high precision and recall, even when a simple ERA is applied instead of the ML clustering.

Our heuristic is a lightweight algorithm based on the *ρ–ratio*. The algorithm's main goal is finding the two sets of records with the highest *ρ–ratio*. Although the algorithm directly applies COROLLARY 3, it is not aligned with the corollary in two aspects:

- *The algorithm may not find the two sets of records with the highest ρ*–ratio: One possibility would be to examine all possible pairs of disjoint sets of records and find the pair with the maximum *ρ–ratio*. However, the computational overhead of this approach makes it infeasible, since the number of such pairs is exponential to the number of records in the dataset. Therefore, our heuristic only examines some of all the possible pairs of record sets.
- *The ρ*–ratio *found, may be less than the* $\frac{(1-p_c^H)}{p_c^H}$ *threshold*: Even if the algorithm finds the pair of record sets with the highest *ρ–ratio*, this ratio may be less than the threshold COROLLARY 3 states. In the case the *ρ–ratio* found is less than the threshold, a question between the two record sets will not formally be a *beneficial* question. Still, such question lies on a part of the dataset where "lack"-of-evidence is detected, as the high *ρ–ratio* points out.

Algorithm 1 DENSE

```
Input: $\mathcal{G}$: The graph of answers
Output: candidates: the pairs of record sets examined by the algorithm
        question: the next question to ask
 1: Pick the pair of records with the highest $\rho$–ratio between them
 2: Add this pair to candidates
 3: while There are more than one sets of records do
 4:     Pick the answer with the highest YES probability between records in different sets
 5:     Merge the two sets of the two records into one
 6:     Compute the $\rho$–ratio between the merged set and every other set
 7:     Add to candidates, the merged set and the set that achieved the highest $\rho$–ratio
 8: end while
 9: Pick the pair of record sets with the highest $\rho$–ratio, from candidates
10: Between these two record sets, randomly pick one record from the first set and one from the second set
11: question := a question between these two records
```

Alg. 1 describes our heuristic. We start with each set of records consisting of a single record and we compute the *ρ–ratio* between every pair of records (lines 1–2). Then, we pick the pair of records having a YES answer with a probability closer to 1.0 than any other pair of records (line 4). We merge the two records into one set and, then, for every other record, we compute the *ρ–ratio* between the merged set and that record (lines 5–6). We continue to merge record sets and compute the *ρ–ratio*s between the currently merged record set and the rest of the record sets, until all records are merged into a single set (lines 3–8). In the end, we select the pair of record sets with the highest *ρ–ratio*, which was found during the process (line 9). The question we select is between two arbitrarily chosen records, one from the first set and one from the second set(lines 10–11). The time complexity of Alg. 1 is $O(n^2)$, where n is the number of records in the dataset.

The rationale behind Alg. 1, is that by merging record sets with strong YES evidence between them, we are less likely to miss a pair of record sets with a high *ρ–ratio* between them. Our choice is related to the fact that *ρ–ratio* is low when YES answers with a high probability are involved. Therefore, we merge sets with "strong" YES answers between them, at each step, so that these "strong" YES answers are not involved in subsequent *ρ–ratio*s examined by the algorithm.

In addition to Alg. 1, which selects a single question to ask next, we propose a batch version of our heuristic in Alg. 2.

Algorithm 2 bDENSE

```
Input: candidates: the pairs of record sets examined by the DENSE
    algorithm
Output: questions: a set of questions to ask next
 1: Sort the pairs of record sets in candidates, in descending order,
    based on the $\rho$–ratio between them
 2: involvedRecs = $\emptyset$
 3: while candidates is not empty do
 4:     Remove the pairs of record sets in the 1st position of
        candidates
 5:     if Any of the records in these two sets is already in
        involvedRecs then
 6:         continue to line 3
 7:     end if
 8:     Between these two record sets, find the pair of records that
        has an answer with a probability closest to 0.5
 9:     Add a question between these two records in questions
10:     Add the records from the two record sets in involvedRecs
11: end while
```

The reason for having a batch version is twofold. First, when we issue multiple questions, we allow many humans to answer questions in parallel. Second, Alg. 1 follows a greedy approach that attempts to find the single best question to ask next. If Alg. 1 finds a pair of record sets with a high *ρ–ratio*, it may keep asking questions between these two record sets, for a large number of consecutive invocations. However, a better strategy may be to explore other record sets of the dataset that may also have a high *ρ–ratio*, and not just focus on the pair with the highest *ρ–ratio*.

In the experimental results of Section VI, we will use bDENSE, the batch version of our heuristic.

V. Entity Resolution Algorithms (ERA)

As discussed in the Introduction, the Entity Resolution Algorithm (ERA) is applied after questions have been posed to the crowd. Based on the evidence collected, the ERA returns a partition of the records, where each cluster represents what is believed to be an entity.

One option is for the ERA to return the Maximum Likelihood clustering. We have argued that the ML clustering is the best guess one can make, so in a sense this solution is the best possible. Unfortunately, finding the ML clustering is prohibitively expensive except in the smallest scenarios. Thus, in practice an ML ERA is not used; instead heuristic approximations are used to find good clusterings.

It is outside the scope of this paper to survey all possible ERAs. Instead we discuss one simple representative algorithm, Transitive Closure (TC). While there are better algorithms (that would take longer to explain), TC represents a big class of algorithms that apply a probability threshold to obtain a deterministic graph of records, which is then partitioned.

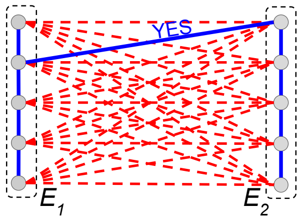
We also present a new ERA that we believe is better suited to partitioning records with the type of evidence we have (without first generating a deterministic graph). While this ERA is a natural extension of TC, as far as we know it has not been discussed in the entity resolution literature.

Note incidentally that our question generation algorithm (Section IV) selects questions that improve the ML clustering (without actually computing the ML clustering). Of course, such questions may not be the best to improve a clustering obtained via a heuristic ERA. Nevertheless, in Section VI we show that our questions are indeed useful even when the heuristic algorithms are used.

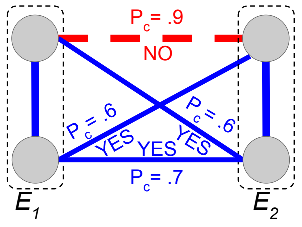
A. Transitive Closure (TC)

The TC algorithm uses a probability threshold to determine which pairs of records refer to the same entity. After identifying such pairs, it applies the transitive relation to connect records into components (clusters). For example, say for a pair of records, a and b, we have a YES with a probability of being correct, higher than the threshold, and the same applies to a pair, b and c. Then we can infer that a and c also refer to the same entity, and hence all three records are in the same cluster.

An issue with TC (and other schemes that apply thresholds) is that it can be misled by erroneous answers. To illustrate, consider the scenario in Fig. 9(a). Entity E_1 consists of 5 records and entity E_2 also consists of 5 records. Therefore, there are 25 pairs of records with one record from E_1 and one record from E_2. Since humans make errors, there is a high likelihood one or more record pairs, out of the 25 pairs, will end up having one or more YES answers and zero NO answers. Just a single pair that happens to have a combined YES probability higher than the threshold will cause TC to incorrectly merge E_1 and E_2. As the number of constituent records increase, the chances that one record pair causes an erroneous merge increase.



(a) Motivation for SCC



(b) SCC merging decision

Fig. 9. SCC description

B. Spectral-Connected-Components (SCC)

The SCC ERA avoids the problem of Fig. 9(a) by only merging clusters like E_1 and E_2 if the combined evidence of all 25 pairs indicates a high probability that the clusters represent the same entity.

SCC starts from the pair of records having the highest probability of being the same entity, given the answers for the two records. If this probability is higher than 0.5, SCC merges the two records into one component (cluster). In each step, SCC finds the pair of components with the highest probability of being the same entity, given the answers between them. If this probability is higher than 0.5 the two components are merged into one. Otherwise, SCC stops merging components, and returns as output the current set of components.

Fig. 9(b) gives an example of how SCC determines if it should merge two clusters or not. SCC examines the probability of E_1 and E_2 being a different entity (event dif), given the

4 answers between E_1 and E_2, denoted by A. Here, $P(dif|A)$ is:

$$\frac{P(A|dif)P(dif)}{P(A|same)P(same)+P(A|dif)P(dif)} = \frac{0.3*0.4*0.4*0.9}{0.7*0.6*0.6*0.1+0.3*0.4*0.4*0.9} \approx 0.63$$

SCC assumes that $P(same) = P(dif)$, i.e., answers involving records that do not belong to E_1 or E_2 are not taken into account for the merging decision. In each step, SCC finds the pair of clusters with the highest $P(same|A)$. If $P(same|A) > P(dif|A)$, SCC merges the two clusters into one. In this example, merging would not take place, since $P(dif|A) > P(same|A)$.

VI. Experimental Results

First, we give a brief overview of the alternative algorithms and the metric used, and then we discuss the experimental settings and our findings.

Question Selection Algorithms: We compare bDENSE (Alg. 2) with HALF, the heuristic proposed in [3], and MLF the Maximum-Likelihood-First strategy proposed in [2]. (Note that a heuristic similar to MLF is proposed in [1].)

HALF picks in each step the most "uncertain" pair of records, i.e., the pair of records with the-closest-to-0.5 probability of being the same entity. Note that the probability of being the same entity, is computed using only local information, i.e., the answers for a specific pair of records. For example, consider *a)* a pair of records with a NO answer and a probability of the answer being correct 0.6 and *b)* a pair of records with a YES answer and a probability of the answer being correct 0.7. HALF will first pick pair *a)* to issue a question.

MLF sorts all pairs of records based on their probability of being the same entity. Again, just like HALF, this is the probability given only the answers for a specific pair of records, and not the global information for the dataset. The MLF strategy starts asking questions from the pair of records with the highest probability of being the same entity, and skips pairs that can be inferred to refer to the same or different entities. The inference is performed via the transitive relations on the human answers collected, up to the current step. For example, if for a pair of records, a and b, we have a YES human answer for this pair, and for a pair, b and c, we also have a YES human answer, then we can infer that a and c also refer to the same entity. In addition, if we get a NO answer for pair b-d, we can infer that a and d refer to different entities, as well as that c and d refer to different entities.

Note that [2] (where MLF is defined) and [3] (where HALF is defined) assume that humans do not make errors. Since in our experiments humans do make mistakes, we extend MLF and HALF in the following way. When MLF or HALF select a pair of records to be evaluated by a human, we actually ask a sequence of humans to evaluate the same pair, until the pair is "fully resolved". To illustrate, assume that for pair of records (a, b), we have received two YES answers and one NO answer, i.e., our local evidence is $y_{ab} \wedge y_{ab} \wedge n_{ab}$. We then compute two probabilities: the probability that a and b are the same entity given this local evidence, and the probability that a and b are different entities again given this evidence. (For this computation we use the expected human accuracy, p_c^H.) If either of these probabilities is greater that some threshold, we say that (a, b) is fully resolved and we stop asking questions for this pair. We use two different thresholds for pair resolution, 0.99 and 0.9999. The corresponding algorithms are HALF99, HALF9999, MLF99, and MLF9999.

Entity Resolution Algorithms (ERA): We use TC and SCC, the ERAs discussed in Section V, in our experiments. In particular, TC is used with MLF and SCC is used in all other cases. The threshold TC uses is the same with MLF.

Metric: We use the F1 score to evaluate the quality of the evidence collected by each question selection algorithm. After a given number of answers collected by a question selection method, we apply the ERA algorithm. Then, we compare the output to the gold standard clustering. Specifically, we compare the pairs of records that refer to the same entity in the output, to the pairs of records referring to the same entity in the gold standard. We get the *precision*(p) and *recall*(r) by comparing the two sets of record pairs and we compute the F1 score, $\frac{2*p*r}{p+r}$.

A. Real Data

In this section, we present our experiments with two datasets of images, *AllSports* [10] and *Gymnastics* [11], a dataset containing CAPTCHAs [12], and a dataset of scientific publications [13].

1) Images Datasets: The *AllSports* [10] dataset contains 267 athlete images from 10 different sports, with each image showing a single athlete. In datasets like *AllSports* and *Gymnastics*, image processing algorithms are still far less accurate than humans. A typical approach for gathering evidence in such cases, is to use a low-cost crowdsourcing platform to gather some initial evidence for every image pair, and then use a smaller pool of trusted annotators to accurately evaluate the critical pairs. We emulate this approach by asking 10 inexperienced workers (no specific requirements) from Amazon Mechanical Turk (*AMT* [8]) to compare each two same-sport images, and generate an initial answer for that image pair. For example, if for a specific pair we get 6 NOs out of 10, we generate an initial NO answer with an associated probability of being correct of $6/10$. (If all workers agree on a pair, we assign a probability of 0.95, as opposed to 1.0, to the YES or NO to reflect the fact the this initial evidence is not really perfect.) For across-sports images, their metadata information is sufficient to produce a machine answer of NO with associated probability 1.0.

In order to answer the questions issued by each question selection algorithm, we build a cache of answers from experienced workers with the following *AMT* qualifications: *a)*at least 97% of their previous tasks in the platform are approved and *b)*at least 500 of their previous tasks are approved. For each two same-sport images we collect 10 answers. In the rare case where question selection algorithms end up asking more than 10 questions on a specific pair, we recycle the answers on that pair once we run out of cached answers.

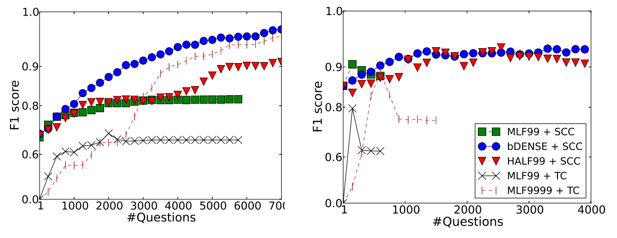
In order to estimate the human accuracy parameter, p_c^H, used by the question selection algorithms, we use a smaller set of

30 images, with 5 "expert" answers per image pair. From these data we compute a p_c^H of 0.9, i.e., 9 out of 10 answers are correct, on average.

The *Gymnastics* dataset [11] contains 94 images of gymnastics athletes. The main difference with the *AllSports* dataset is that *Gymnastics* contains images where it is very difficult to distinguish the face of the athlete, e.g. the athlete is upside down on the uneven bars.

To construct the initial answers, we follow the same procedure with the one described for *AllSports*. The only difference is that for each image pair we ask fewer questions, since we need to ask questions for more pairs compared to *AllSports* (in *Gymnastics* we ask questions between any two images, as opposed to *AllSports* where we ask questions only between same-sport images). In particular, we ask for each pair 5 questions to *AMT* workers without qualifications and 5 questions to workers–"experts".

Using a training set of 30 images, we compute a human accuracy parameter, p_c^H, of 0.9; the same as in *AllSports*.



(a) *AllSports* (b) *Gymnastics*

Fig. 10. F1 score for the two images datasets

Fig. 10(a) depicts the performance of the five approaches for the *AllSports* dataset. The Y-axis gives the F1 score on an exponential scale, as different approaches issue new questions (X-axis), until we reach a limit of 7000 questions. Each curve shows the performance under a question selection algorithm and an ER algorithm. For example, in the case of bDENSE+SCC, bDENSE selects the questions and SCC performs the resolution.

The solutions that use SCC start (at 0 questions) with an F1 score of about 0.7, yielded by the initial evidence. As questions are asked, bDENSE improves the F1 score more rapidly than the other schemes, as it selects pairs for questions more intelligently. For instance, HALF99+SCC requires twice as many questions as bDENSE+SCC to reach an accuracy of 0.9. MFL99+SCC flattens out and does not ever reach 0.9 accuracy. There are two reasons why MFL99+SCC flattens out and remains below an F1 of 0.9. The first reason is that MLF was designed to work with the TC ERA. That is, when MLF selects the questions to ask next, it will skip questions whose answer can be inferred through transitive relations. However, when SCC is performing the resolution such questions may be very useful. The second reason is that with this data set MLF99+SCC gets confused by certain "problematic pairs" where humans often provide the wrong answer. (We discuss these problematic pairs more below.) MLF99 repeats each question until it thinks that it has the right answer (with probability at least 0.99). In spite of this repetition, it often gets the incorrect answer for a problematic pair of images, leading to a poor overall clustering.

MLF9999+SCC (not shown in Fig. 10(a) to avoid clutter) repeats each question until it believes it has the correct answer with probability at least 0.9999, reducing the impact of problematic pairs, but consuming more questions in the process. The net result is that MLF9999+SCC starts "slower" than MFL99+SCC, however, after 2500 questions it surpasses MFL99+SCC and stays very close to MLF9999+TC.

Fig. 10(a) also shows the performance of the solutions that use TC for entity resolution. Initially TC achieves a 0.0 F1 score because it does not use the initial evidence. (With the initial machine evidence, many pairs do not have an above threshold probability, so using transitive closure at that point is not productive [2].) Thus, MLF needs to ask many more questions to improve accuracy. We observe a similar behavior as before: MLF99+TC gets confused by problematic pairs and never goes above an F1 of 0.65; MLF9999+TC performs better but needs to ask more questions due to its repetitions.

The results for the *Gymnastics* dataset are shown in Fig. 10(b). The *Gymnastics* dataset is a much tougher one, as it contains significantly more problematic pairs. The net result is that *(1)* none of the SCC schemes can improve accuracy much beyond the initial F1=0.85 value, and *(2)* even MLF9999+TC gets confused and flattens out roughly at F1=0.75.

We have argued that one of the key performance factors is the number of problematic pairs. The *Gymnastics* dataset contains more of them because athletes are often photographed upside down, grimacing or with hair covering their face. Fig. 11 quantifies our notion of problematic pairs for each dataset. The figure shows the percentage of pairs (Y-axis, log scale) that have a given percentage of wrong answers (X-axis). For example, for the *Gymnastics* set we ask 5 questions for each pair; if we get one correct and 4 wrong answers we have an 80% wrong answer percentage (X-axis), and Fig. 11 tells that about 1% of the pairs were in this category. Note that for the *AllSports* set where we ask 10 questions, there are 11 possible X-axis values, as opposed to only 6 for Gymnastics.

In addition, note that when the percentage of wrong answers is higher than 0.5, we get an overall wrong answer for the pair if we keep repeating the same question. These pairs are problematic and will cause the algorithms to cluster records incorrectly. Fig. 11 shows that *Gymnastics* has more problematic pairs; the absolute numbers are not large, but it does not take many problematic pairs to confuse the resolution algorithms. The effect is more pronounced when TC is used for resolution, since a single incorrect pair can cause clusters to join or break up.

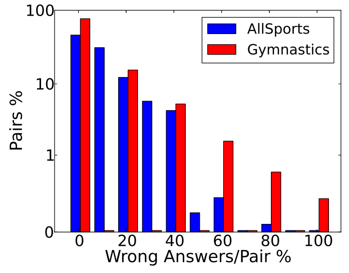


Fig. 11. Distribution of human errors on image pairs.

2) CAPTCHAs Dataset: The CAPTCHAs [12] dataset consists of 244 CAPTCHA images, each showing a four-digit number. The number of records per entity follows a power-law distribution with an exponent of -2.5. We produce initial answers between pairs of images, using a simple image processing algorithm we implemented, which computes the probability distribution for each digit. For example, for the first digit, we compute the probability of the digit being a "0", the probability of being a "1", and so on. Combining the distributions for the four digits of two images, we compute an initial answer along with a probability of the answer being correct. Human answers are provided on demand to each question selection algorithm, by a cache of answers we constructed using AMT. In particular, for each of the 29646 pairs we ask 5 questions and store the answers in the cache. In order to explore a scenario with "noisy" human answers we did not set any specific requirements on the workers processing the tasks. Using a smaller training set of CAPTCHAs, we compute a human accuracy, p_c^H, of 0.7. This value is lower than for the images datasets, mainly because we do not use any worker requirements here.

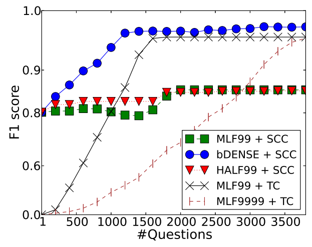


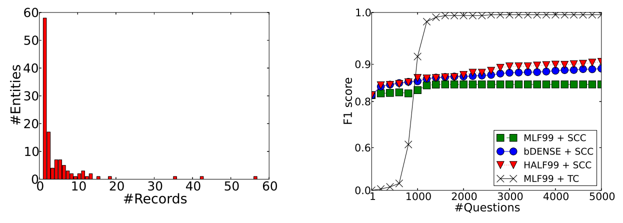
Fig. 12. *CAPTCHAs*: F1 score.

Fig. 12 shows the results for CAPTCHAs. There are two differences compared to the results for the images datasets. First, bDENSE needs even fewer questions to converge to a highly accurate outcome. (Note that the total number of pairs we can question is larger than in *Gymnastics* and *AllSports*, yet high accuracy is obtained with fewer human questions.) Second, MLF99 converges much faster to high accuracy compared to MLF9999.

There are two key factors explaining these differences. First, because of the records/entity power-law distribution used in the CAPTCHAs data set, there are a lot of entities with more than two or three records. It turns out that large entities can "magnify" the impact of question selection. When large entities are correctly identified, the F1 score improves a lot (since many record pairs make up the entity); if mistakes are made, the F1 can significantly suffer. Fig. 12 shows that since bDENSE+SCC asks questions intelligently, with respect to the overall evidence, it quickly finds large entities and the F1 score is boosted. The other algorithms under SCC do not rely that heavily on the machine evidence and the net result is that their F1 score flattens out. (As discussed earlier MLF was not designed to work with SCC and the large entities, here, amplify that weakness.) Note incidentally that the impact of large entities has also been observed in [2] and [1]. Also note there are cases (see below) where large entities amplify bDENSE's *errors* (e.g., when the initial evidence is poor).

The second key factor is that there are very few problematic pairs in the CAPTCHAs data set; almost all pairs have a majority of correct human answers. Because there are few problematic pairs, the higher threshold of 0.9999 for MLF9999+TC does not give any benefits, but instead slows down the acquisition of answers. For instance, in Fig. 12 MLF9999+TC needs to ask twice as many questions to reach an F1 of 0.95 compared to MLF99+TC. Still, bDENSE clearly outperforms both methods by reaching an F1 of 0.9 using almost half of the questions needed by MLF99+TC.

3) Cora: We use the real dataset *Cora* [13], and we generate initial answers using the Jaro similarity function [14]. More precisely, we use the title and author fields from each record, and for a pair of records we compute the average Jaro similarity of the title and author. We then use a training set of 300, arbitrarily picked, records, to generate a mapping from similarity to probability of being the same entity, just as in [3]. For pairs of records with a probability of being the same entity, P, less than 0.5, we generate a NO answer with probability of being correct $1 - P$. Then, we arbitrarily pick 500 records as our testing set. In Fig. 13(a), we plot the number of entities with a specific number of records per entity, for the 500 records of our testing set. Every time each approach selects a question to ask next, we generate a human answer synthetically, using an *HA* of 0.9.



(a) Records Per Entity (b) F1 for an *HA* of 0.9

Fig. 13. *Cora* dataset

Fig. 13(b), depicts the F1 accuracy for the 4 approaches, until each approach asks a total of 5000 questions. We see a trade-off appearing, between *a)* getting a high F1 score with a few questions but not "fully resolving" the dataset after 5000 questions, in bDENSE+SCC and HALF99+SCC, and *b)* starting "slowly" but reaching eventually an F1 of 1.0, in MLF99+TC. In particular, bDENSE+SCC and HALF99+SCC start from an F1 of 0.8 and reach 0.85 after 1000 questions. At this point, MLF99+TC gets ahead from the SCC methods and, then, after 1500 questions achieves a full resolution with an F1 of 1.0. On the other hand, bDENSE+SCC and HALF99+SCC converge to an F1 close to 0.9 after 5000 questions.

Let us now analyze the reason why bDENSE+SCC does not converge to an F1 of 1.0. We will refer to two specific entities from the dataset, in order to describe the case. Entity tagged as *fahlman1990a* has 10 records in our testing set, while entity tagged as *fahlman1990b* has 42 records. The records from both entities have the same title and authors; with one entity being the technical report, and the other entity being the corresponding conference paper. Therefore, each of the 10 records of *fahlman1990a* have a "strong" initial YES answer with each of the 42 records of *fahlman1990b*. For this reason, bDENSE can not detect a "lack"-of-evidence between the records of *fahlman1990a* and *fahlman1990b*, and does not issue questions between the two record sets; questions required for the resolution algorithm to infer that these records refer to different entities. The same case with the one taking place between the records of *fahlman1990a* and *fahlman1990b*, happens frequently in the dataset, when we use only the title and author fields, causing the low F1 for bDENSE+SCC.

To summarize our findings so far:

- bDENSE shows the best performance in three out of the four datasets discussed in this section. Note also that in each of those three dataset there was a different second-best approach; MLF9999+TC in *AllSports*, HALF99 in *Gymnastics*, and MLF99+TC in CAPTCHAs. Therefore, the other schemes appear to be less "dependable" compared to bDENSE.
- All schemes are adversely affected by poor machine and/or human evidence, and they react differently to such poor evidence. In general, we found that bDENSE can handle poor evidence better than the other schemes. However, there are cases where the initial evidence is so poor that bDENSE's "intelligence" backfires and yields worse results than the other schemes. In particular, we have observed two situations (one with CAPTCHAs with a "misleading" image processing algorithm, and the one discussed with the *Cora* dataset) where bDENSE performs worse than the other schemes.
- Large entities (with many records) can "amplify" the differences between schemes, making bDENSE look more attractive when initial evidence is reasonably good.
- The MLF schemes are sensitive to the number of repetitions needed to mask out human errors. If the probability threshold used by MLF is chosen incorrectly, questions may be wasted, or errors can be made. Choosing the right threshold seems to be a difficult problem, because it requires thorough knowledge of the dataset and statistics about the human accuracy on the specific dataset (e.g., differences between *AllSports* and *Gymnastics*).

B. Synthetic Data

In addition to the experiments on real data described in the previous section, we also built a simulator that generates synthetic data, and mimics the machine and human record comparisons that need to be performed. The simulator lets us study our algorithms in a controlled environment, where we can clearly identify the factors that affect performance. Furthermore, the simulator makes it possible to study a much broader set of scenarios, and with its visualization interface, it lets us view the algorithms in action.

We start by describing the dataset generation process and the main synthetic data parameters and, then, we discuss our findings for different scenarios. The details of the generation process are given in Alg. 3.

First, we generate a dataset of *RECs* records, where the number of records per entity follows a distribution *DISTRO*. In addition, entities are grouped into buckets. Parameter *BUCKET* gives the total number of buckets used. The entities in one bucket can have records that look similar, while records are assumed quite dissimilar to those in other buckets.

In particular, the initial evidence for record r compared to record s is generated as follows. If s is in a different bucket than r, the initial evidence is NO (records do not match) with a reported confidence of 1.0. If r and s are part of the same entity, we pick a value c between 0.5 and 1 to represent the confidence of a machine comparison. Then with probability c the initial evidence is correct (YES), and with probability $1-c$ it is incorrect (NO). In either case we associate a confidence of c with the answer. If r and s are in the same bucket but not part of the same entity, with probability M_A we say that r, s are similar, and with probability $1 - M_A$ we say they are different. If the records are different, then the initial evidence says NO with a confidence of 1.0. If r, s are similar we again generate a match probability c between 0.5 and 1, and then report (confidence c) correct evidence (NO) with probability c and an incorrect YES with probability $1 - c$.

For human comparisons, we assume an accuracy H_A: with probability H_A we report a correct answer (confidence H_A) and with probability $1 - H_A$ we report an incorrect answer.

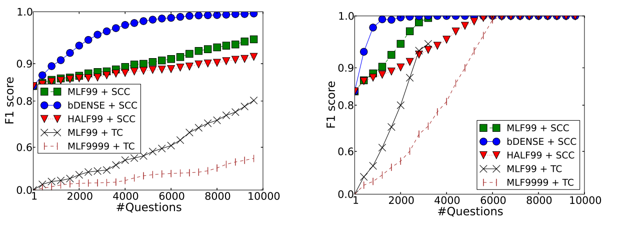
To simulate what we have called problematic pairs, we define a parameter *PROBLEMATIC* that specifies the fraction of pairs of records of the same entity that are problematic. That is, for each entity we select at random a fraction *PROBLEMATIC* of its possible pairs and label them as problematic. Then we change the rules (given above) for comparing records r and s of the same entity as follows: If r, s is a problematic pair, flip the answer (i.e., with probability $c \in [0.5, 1.0]$ the initial evidence is *incorrect*, or with probability H_A the human answer is *incorrect*.)

Table I summarizes all the parameters used in the synthetic data experiments.

TABLE I
SYNTHETIC DATA PARAMETERS

Parameter	Description	Values	Base Case
RECS	Total Dataset Records	$\{500\}$	500
DISTRO	Gold-Standard Records-Per-Entity Distribution	{GAUSSIAN, POWERLAW}	GAUSSIAN
BUCKET	Bucketing Factor	$\{20\}$	20
M_A	Machine Accuracy	$\{0.9, 0.6\}$	0.9
H_A	Human Accuracy	$\{0.7, 0.9\}$	0.7
PROBLEMATIC	Controls problematic pairs	$\{0.0, 0.2\}$	0.0

In order to generate each curve in the plots in this section, we generate 3 different gold clusterings, and for each clustering we repeat the experiment 4 times. The curve for each approach is the average over the 12 runs.



(a) Faulty-workers, $H_A = .7$ (b) Reliable-workers, $H_A = .9$

Fig. 14. High quality initial answers ($M_A = .9$).

For the base case in Table I, Fig. 14(a) gives the F1 score, as different approaches issue new questions, until we reach a limit of 10000 questions. Note that in the base case we use a Gaussian distribution for the number of records per entity (DISTRO), with a mean of 3.0 and a variance of 2.0.

Note that the results in Fig. 14(a) show the same general relationships between algorithms as Fig. 10(a) (*AllSports* data set). That is, bDENSE outperforms the other strategies, and as more questions are asked, the other strategies catchup, albeit at different rates. The F1 values in Fig. 14(a) are higher than before, mainly because there are no irregularities (e.g., problematic pairs).

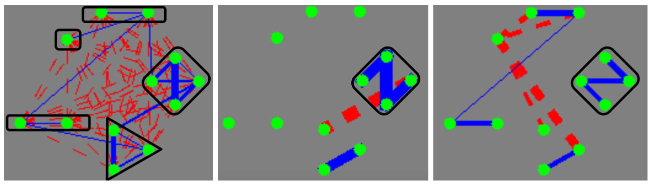


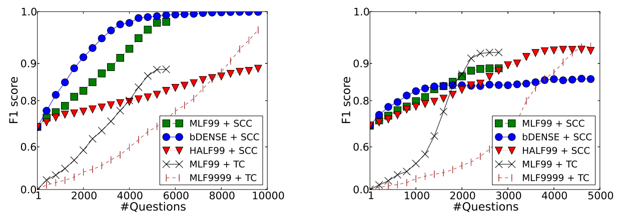
Fig. 15. Simulator screenshots comparing MLF and bDENSE question selection.

The reasons why bDENSE performs well compared to the other strategies were discussed in Section VI-A. Now with our simulator we can confirm those observations by seeing how particular sets of records are resolved. To illustrate, Fig. 15 shows how 4 given entities are resolved. In this case, the generator created one entity with 4 records, one entity with 3 records, two entities with 2 records, and one entity with a single record. On the left side of the figure, we see the initial answers, used as input by all approaches, along with the generated clustering. We use green dots for records, dashed red lines for NOs, and solid blue lines for YESes. The thickness of each line is proportional to the probability of the combined answer for the two corresponding records. In the middle, we see how MLF chooses to spend 20 questions, while on the right side, we see how bDENSE spends 20 questions. MLF chooses to "fully" resolve the relation between 5 pairs of records, while bDENSE distributes the 20 questions on 11 pairs of records. Instead of trying to build large connected components like MLF, bDENSE does not insist on certain pairs of records and prioritizes pairs based on the "lack"-of-evidence it detects. For example, note that inside the entity of 4 records, bDENSE asks questions on 3 record pairs with "weak" initial answers (thin edges in the left side of the figure), while MLF asks questions on 3 record pairs with "strong" initial answers (thick edges in the left side of the figure).

In Fig. 14(b), we examine a scenario with more "reliable" workers. Thus, we keep the same parameters with the base case, however, we change human accuracy, H_A, from 0.7 to 0.9. All five approaches converge much faster compared to the base case of Fig. 14(a). This effect is analogous to what we observed in Fig. 12 (CAPTCHAS data set). Again, bDENSE+SCC is considerably better than the other approaches, and needs 75% fewer questions than MLF99+SCC to reach 0.95, and 85% fewer questions than HALF99+SCC.

Next, we examine the effect of lowering the quality of the initial answers. Hence, we decrease the machine accuracy, M_A, from 0.9 to 0.6. Human accuracy, H_A, is still 0.9 and the rest of the parameters remain the same. As we see in Fig. 16(a), bDENSE+SCC once again outperforms the other approaches, requiring 40% fewer questions to reach $F_1 = 0.9$, compared to the second best, MLF99+SCC. Even with a machine accuracy of 0.6, there is enough information in the initial evidence to let bDENSE select reasonable initial human questions, and since human answers are high quality, the F1 score improves rapidly.

In order to create a scenario where bDENSE does worse than some of the other approaches, we need to change two additional parameters. First we simulate problematic pairs by setting *PROBLEMATIC* to 20%. This means that in 20% of the cases where we ask a human to compare records that are part of the same entity, they will give us the wrong answer with high probability ($H_A = 0.9$). (The initial evidence will be similarly biased.) Second, we use a POWERLAW *DISTRO* with an exponent of -3.0. This distribution gives us some large entities, which as we discussed earlier amplify the impact of question selection. The rest of the parameters remain the same as in the base case, except for human accuracy which is 0.9 as in our previous experiment.



(a) Low Machine Accuracy ($M_A = .6$)
(b) POWERLAW with problematic pairs

Fig. 16. Low quality initial answers.

As we see, in Fig. 16(b), the performance of all four approaches drops significantly. The effect is more marked with bDENSE+SCC. In this situation, bDENSE's attempt to select questions to ask intelligently backfires since it is based on incorrect evidence. And this poor performance is magnified when large entities are incorrectly resolved. On the other hand, HALF, does not take into account the graph structure, and ends up asking relatively better questions. Similarly, even MLF99+TC eventually achieves a better F1 value by being conservative and asking and re-asking questions that already have relatively good evidence.

The scenario of Fig. 16(b) simulates and explains the behavior we observed with the *Cora* data set, discussed in Section VI-A.

VII. Related Work

Entity Resolution is a well-studied problem and goes by various names including record linkage, deduplication, identity resolution, object identification, merge/purge, reference reconciliation. Surveys [15] and [14] discuss approaches and algorithms proposed for Entity Resolution.

Recently, a number of approaches have been proposed for enhancing ER with evidence provided by humans [16], [7], [17], [2], [3], [1], [18]. Reference [17] proposes an interface where a human sees more than two records and tags records representing the same entity with the same label. The main objective in [17] is the "efficient packaging" of records into tasks for humans. A similar interface with more than two records in each human task is used in [16]. The main focus of [16] is the probabilistic aggregation of human tasks' outcome into an overall clustering.

As opposed to [16] and [17], references [2], [3], [1], [7] use a simpler interface for the human task, involving only two records. All of [2], [3] and [1], propose question selection algorithms for ER, having as their main assumption that humans do not make errors. On the contrary, our approach assumes that humans are not always correct. The same assumption is also made by [7], however, the main focus of [7] is designing an Entity Resolution Algorithm (ERA) that can handle human errors. The question selection process of [7] just randomly selects the record pair to resolve at each step; once the two records are selected the paths connecting them are examined to find the question on the edge that will reveal the "most" about the relationship of the records.

An approach complementary to our approach is proposed in [18], where crowdsourcing is applied in additional steps of an ER workflow, e.g., by using active learning to train the similarity function producing the initial (machine) answers.

References [19], [20], [21] study similar problems to the question selection for ER. Reference [19] defines the "most informative" question as the question that will cause the "biggest" change in the current clustering, after the answer for that question is retrieved. References [20] and [21] focus on spectral clustering [22], and try to find the questions that will enhance the "most" the evidence used for spectral clustering. While spectral clustering can be used for a k-way partitioning of records into clusters, the approaches proposed in [20] and [21] focus in the 2-way partitioning case. As opposed to [19], [20], [21], our definition for the "most beneficial" question, along with our results and algorithm, are based on a Maximum Likelihood formulation.

The Maximum Likelihood clustering is closely related to the problem of correlation clustering [9]. In fact, the ML clustering, presented in Section II-B, can be reduced to the weighted version of correlation clustering, as [23] discusses.

VIII. Future Work

There are a number of possible extensions on the problem setting that we have not considered in this paper. Here, we describe two of them and, in addition, we discuss three more topics for future work. We believe that these five directions constitute the most promising ground to further improve the efficiency of our approach.

In the first extension, we can take into account prior knowledge about the distribution of records referring to the same entity, e.g., power-law or gaussian. As we discussed in our experimental evaluation, there are cases where our approach could greatly benefit from such knowledge. As a second extension, we can consider a different probability of a human making an error for each pair of records. That is, we can estimate the error rate on a specific pair of records based on the answers we get back for this pair, e.g., if the YES/NO answers are balanced, the error rate must be "high" for that pair.

In the experimental evaluation, we mentioned that our approach is sensitive on the initial evidence, in some cases. The thorough study for the effect of the initial answers' quality to our approach's effectiveness is an interesting direction for future work. Another direction is the study of graph algorithms that try to find the pair of subgraphs with the highest *ρ–ratio* between them. In this paper, we proposed a simple heuristic for finding disjoint sets with a high *ρ–ratio*, however, more sophisticated algorithms can give a better outcome and, potentially, improve the overall accuracy of our approach. Finally, it would be interesting to explore the effect of other ERA, when applied using the evidence that bDENSE collects. For example, an ERA with performance closer to the ML clustering could yield even better results than SCC.

IX. Conclusion

We studied the problem of enhancing Entity Resolution using human answers that may possibly be erroneous. The key challenge is how to reason about likely record clusters (entities) when both the computer generated and human generated evidence can be incorrect. We derived theoretical results that help us identify beneficial questions to ask humans, i.e., questions whose answers can improve the accuracy of the Maximum Likelihood clustering. These insights lead to a practical and efficient algorithm (bDENSE) for selecting questions to ask. Through our experiments we showed that this algorithm often outperforms other approaches, even when heuristic clustering algorithms (ERAs) are used to perform the final record partition.

We also discovered that ERAs that apply simple thresholds do not handle erroneous evidence well. Instead we suggested a Spectral Connected Components (SCC) ERA that only merges two clusters when the overall evidence indicates that it is likely that the two sets of records represent the same entity. Our experiments indicate that this SCC ERA outperforms other approaches (unless computer or human evidence is very misleading).

References

[1] N. Vesdapunt, K. Bellare, and N. Dalvi, "Crowdsourcing algorithms for entity resolution," in *PVLDB*, 2014.

[2] J. Wang, G. Li, T. Kraska, M. J. Franklin, and J. Feng, "Leveraging transitive relations for crowdsourced joins," in *SIGMOD*, 2013.

[3] S. E. Whang, P. Lofgren, and H. Garcia-Molina, "Question selection for crowd entity resolution," in *PVLDB*, 2013.

[4] P. G. Ipeirotis, F. Provost, and J. Wang, "Quality management on amazon mechanical turk," in *Proc. of the ACM SIGKDD Workshop on Human Computation (HCOMP)*, 2010.

[5] A. G. Parameswaran, H. Garcia-Molina, H. Park, N. Polyzotis, A. Ramesh, and J. Widom, "Crowdscreen: Algorithms for filtering data with humans," in *SIGMOD*, 2012.

[6] P. Venetis and H. Garcia-Molina, "Quality control for comparison microtasks," in *Proc. of the Int. Workshop on Crowdsourcing and Data Mining (CrowdKDD)*, 2012.

[7] A. Gruenheid, D. Kossmann, S. Ramesh, and F. Widmer, "Crowdsourcing entity resolution: When is a=b?" *Technical report, Systems Group, Department of Computer Science, ETH Zurich*, 2012.

[8] "Amazon mechanical turk," https://www.mturk.com.

[9] N. Bansal, A. Blum, and S. Chawla, "Correlation clustering," in *FOCS*, 2002.

[10] "Allsports," stanford.edu/~verroios/datasets/allsports.zip.

[11] "Gymnastics," stanford.edu/~verroios/datasets/gymnastics.zip.

[12] "Captchas," stanford.edu/~verroios/datasets/captchas.zip.

[13] "Cora dataset," people.cs.umass.edu/~mccallum/data/cora-refs.tar.gz.

[14] W. Winkler, "Overview of record linkage and current research directions." *Technical report, Statistical Research Division, U.S. Bureau of the Census, Washington, DC*, 2006.

[15] A. K. Elmagarmid, P. G. Ipeirotis, and V. S. Verykios, "Duplicate record detection: A survey." *IEEE Trans. Knowl. Data Eng.*, vol. 19, no. 1, pp. 1–16, 2007.

[16] R. Gomes, P. Welinder, A. Krause, and P. Perona, "Crowdclustering," in *NIPS*, 2011.

[17] J. Wang, T. Kraska, M. J. Franklin, and J. Feng, "Crowder: Crowdsourcing entity resolution," in *PVLDB*, 2012.

[18] C. Gokhale, S. Das, A. Doan, J. F. Naughton, N. Rampalli, J. Shavlik, and X. Zhu, "Corleone: Hands-off crowdsourcing for entity matching," in *SIGMOD*, 2014.

[19] A. Biswas and D. W. Jacobs, "Active image clustering: Seeking constraints from humans to complement algorithms," in *CVPR*, 2012.
[20] O. Shamir and N. Tishby, "Spectral clustering on a budget," in *Proc. of the Int. Conf. on Artificial Intelligence and Statistics (AISTATS)*, 2011.
[21] F. L. Wauthier, N. Jojic, and M. I. Jordan, "Active spectral clustering via iterative uncertainty reduction," in *KDD*, 2012.
[22] J. Shi and J. Malik, "Normalized cuts and image segmentation," *Trans. on Pattern Analysis and Machine Intelligence*, vol. 22, pp. 888–905, 1997.
[23] S. Bagon and M. Galun, "Large scale correlation clustering optimization," *CoRR*, 2011.

Appendix

Algorithm 3 Synthetic Dataset Generation

```
Input: RECs: The number of records in the dataset
       DISTRO: The distribution controlling the number of records per entity
       BUCKET: How many entities to place in each bucket
       $M_A$: Machine Accuracy
Output: Gold standard clustering and
       one initial answer for each pair of records
 1: {Entities Generation}
 2: while the number of records generated are less than RECs do
 3:   Using DISTRO generate a number $n$
 4:   Create a new entity with $n$ records
 5: end while
 6: {Buckets Generation}
 7: while there one or more entities left do
 8:   Randomly pick BUCKET entities
 9:   Group the selected entities' records into a bucket
10:   Remove the selected entities from the set of all entities
11: end while
12: {Initial Answers Generation}
13: for each bucket do
14:   for each pair of records inside the bucket do
15:     if the two records refer to the same entity then
16:       Pick uniformly a probability $p$ in $[0.5, 1.0)$
17:       Pick uniformly a number $x$ in $[0.5, 1.0)$
18:       if $x < p$ then
19:         Create a YES answer with $p$ attached
20:       else
21:         Create a NO answer with $p$ attached
22:       end if
23:     else
24:       Pick uniformly a number $x$ in $[0.5, 1.0)$
25:       if $x > M_A$ then
26:         Pick uniformly a number $x$ in $[0.5, 1.0)$
27:         if $x > \epsilon$ then
28:           Create a NO answer with $1 - \epsilon$ attached
29:         else
30:           Create a YES answer with $1 - \epsilon$ attached
31:         end if
32:       else
33:         Pick uniformly a probability $p$ in $[0.5, 1.0)$
34:         Pick uniformly a number $x$ in $[0.5, 1.0)$
35:         if $x < p$ then
36:           Create a NO answer with $p$ attached
37:         else
38:           Create a YES answer with $p$ attached
39:         end if
40:       end if
41:     end if
42:   end for
43: end for
44: for each pair of records between buckets do
45:   Pick uniformly a number $x$ in $[0.5, 1.0)$
46:   if $x > \epsilon$ then
47:     Create a NO answer with $1 - \epsilon$ attached
48:   else
49:     Create a YES answer with $1 - \epsilon$ attached
50:   end if
51: end for
```